MAJ-003-001513 Seat No. _____

B. Sc. (Sem. V) (CBCS) Examination

October / November – 2016

Mathematics : Paper - BSMT - 501 (A)

(Mathematical Analysis & Group Theory)

Faculty Code : 003

Subject Code : 001513

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 Answer the following Questions in one word : **20**

(1) $f(x)=\frac{20}{x}, x\in[2,20]$ and $P=\{2,4,5,20\}$. Find $U(P,f)$.

(2) State Darboux's theorem.

(3) State General form of First Mean Value Theorem.

(4) In usual Notation, Formula of Second Mean Value Theorem of Integral Calculus of weiestrauss form is _____.

(5) In usual Notation, $\underline{\int} f(x)dx$ =

(6) If $E=[3,5]$ is subset of metric space R then $E°$ = _____.

(7) If E = (1, 5) is subset of metric space R then E' = _____.

(8) The border set of the subset E = (1, 2) of metric space R is _____.

(9) The derive set of subset (4, 9) of metric space R is _____.

(10) Define Limit Point.

(11) Give an example of Group which not abelian.

(12) Define Automorphism.

(13) Define Normal Subgroup.

(14) Define Order of Group.

(15) Find generators of Cyclic Group $(Z_6,+_6)$.

(16) Define Right Coset.

(17) Define Isomorphism of Group.

(18) Define Factor Group.

(19) Examine whether the following permutation is even or odd $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 3 & 5 & 2 & 8 & 6 & 1 & 7 \end{pmatrix}$.

(20) Find $O(f)$ where $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 2 & 9 & 4 & 6 & 1 & 3 & 5 & 8 \end{pmatrix}$

2 (a) Answer any **three** : **6**

(1) Prove that the function f is continuous in $[a, b]$ then $f \in R_{[a, b]}$.

(2) State and prove First Mean Value theorem of Integral Calculus.

(3) Prove that $\int_{-}^{-} f(x)dx \leq \int^{-} f(x)dx$

(4) Prove that finite Union of finite number of closed set of metric Space Is closed set.

(5) If (X, d) is metric space and $A, B \subset X$ then $A \subset B \Rightarrow A' \subset B'$.

(6) Give an example of subsets A and B of a metric space R such that $(A \cap B) = \bar{A} \cap \bar{B}$.

(b) Answer any **three** : **9**

(1) For $0 < a < b$ prove that

$$\frac{\Pi^2}{2a} \leq \int_0^{\Pi} \frac{x^2 dx}{a\cos^2\frac{x}{2} + b\sin^2\frac{x}{2}} \leq \frac{\Pi^2}{2b}$$

(2) Using definition of R – Integration Evaluate $\int_0^a x^2 dx$.

(3) State and Prove Fundamental theorem of Integration.

(4) Determine the subset $E_n = \left(\frac{1}{n}, \frac{n-1}{n}\right)$ of metric space R is open or closed.

(5) Prove that Every finite subset of any metric space is closed.

(6) Check following function defined on R is Metric space or Not $\forall x, y \in R, d(x, y) = \left|x^2 - y^2\right|$.

(c) Answer any **two** : **10**

(1) Let f be bounded function on $[a, b]$ then necessary and sufficient condition for $\int_a^b f(x)dx = \int_a^{\overline{b}} f(x)dx = \int_{\underline{a}}^b f(x)dx$ is that $\lim_{\|P\|\to 0} S(P, F)$ exists and value of this limit is $\int_a^b f(x)dx$.

(2) Evaluate $\lim_{n\to\infty}\left[\frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots + \frac{1}{2n}\right]$

(3) Prove that the closer of a subset of metric space is a closed set.

(4) Prove that $\frac{3}{4}$ is in cantor set.

(5) Prove that (R, d) is a separable metric space.

3 (a) Attempt any **three** : **6**

(1) Let $(G, *)$ be a group, then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$, $\forall a, b \in G$.

(2) If Binary operation * defined as $a*b = ab+1$, $\forall a, b \in G$, Is $(G, *)$ group or not ?

(3) Show that every group with identity 'e' such that $a^2 = e, \forall a \in G$ is abelian.

(4) Let $H \leq G$ and $a, b \in G$, if $H_a = H_b$ then prove that $ab^{-1} \in H$.

(5) If $f=\begin{pmatrix}1&2&3&4&5&6\\6&5&3&1&2&4\end{pmatrix}$ and

$g=\begin{pmatrix}1&2&3&4&5&6\\5&3&2&4&6&1\end{pmatrix}$ then find fg and gf.

(6) Prove that every cyclic group is abelian.

(b) Attempt any **three** : **9**

(1) Using Fermat's theorem, find the remainder when 8^{103} is divided by 13.

(2) If $H \leq G$ then show that $x^{-1}Hx=\left\{x^{-1}hx / h\in H\right\}$ is also subgroup of G; $\forall x \in G$.

(3) Let G be a group and $H=\left\{a^2 / a\in G\right\}\leq G$, Then show that H is normal subgroup of G.

(4) Show that a non empty subset H of group G is subgroup of G iff $ab^{-1}\in H,\ \forall a,b\in H$.

(5) Let $H\leq G$ and $K\leq G$ Then Prove that $K\bigcap H$ is normal subgroup of K if H is normal subgroup of G.

(6) Show that the mapping $f:(R,+)\rightarrow\left(R^+,\times\right)$ defined by $f(x)=e^x, \forall x\in R$ is an isomorphism.

(c) Attempt any **two** : **10**

(1) State and Prove Lagrange's Theorem.

(2) State and Prove Cayley's Theorem.

(3) Define Alternating Group A_n, Show that $A_n(n\geq 2)$ is a subgroup of S_n of order $\frac{n!}{2}$.

(4) Prove that every subgroup of cyclic group is cyclic.

(5) A subgroup H of group G is normal subgroup iff $(H_a)(H_b)=H_{ab};\ \forall a,b\in G$.